Day 12: Inference for a single proportion or difference of two (independent) proportions (Sections 8.1-8.2)

BSTA 511/611

Meike Niederhausen, PhD
OHSU-PSU School of Public Health

2023-11-08

MoRitz's tip of the day: code folding

- With code folding we can hide or show the code in the html output by clicking on the Code buttons in the html file.
- Note the </> Code button on the top right of the html output.

```
▼ Code

2*pnorm(-0.3607455)

[1] 0.7182897
```

```
▶ Code

[1] 0.7182897
```

```
---
title: "Day 12: Inference for a single proportion or difference of two (independent) proportions (Sections 8.1-8.2)"
subtitle: "BSTA 511/611"
author: "Meike Niederhausen, PhD"
institute: "OHSU-PSU School of Public Health"
date: "11/8/2023"
categories: ["Week 7"]
format:
  html:
    link-external-newwindow: true
    toc: true
    code-fold: show
    code-tools: true
    source: repo
execute:
  echo: true
  freeze: auto  # re-render only when source changes
# editor: visual
editor_options:
  chunk_output_type: console
---
```

show code initially shown
true code initially hidden

Creates button at top right of html output that lets the user select: Hide All Code, Show All Code, or View Source

Can specify location of source file for View Source user option. Since this file is stored in a GitHub repository, I specified repo.

Day 12: Inference for a single proportion or difference of two (independent) proportions (Sectio[ns] 8.1-8.2)

BSTA 511/611

WEEK 7

</> Code ▾
- Show All Code
- Hide All Code
- View Source

See more information at https://quarto.org/docs/output-formats/html-code.html#folding-code

Where are we?

CI's and hypothesis tests for different scenarios:

$$\text{point estimate} \pm z^*(or\ t^*) \cdot SE, \quad \text{test stat} = \frac{\text{point estimate} - \text{null value}}{SE}$$

Day	Book	Population parameter	Symbol	Point estimate	Symbol	SE
10	5.1	Pop mean	μ	Sample mean	$\bar{x}$	$\frac{s}{\sqrt{n}}$
10	5.2	Pop mean of paired diff	μ_d or δ	Sample mean of paired diff	$\bar{x}_d$	$\frac{s_d}{\sqrt{n}}$
11	5.3	Diff in pop means	$\mu_1 - \mu_2$	Diff in sample means	$\bar{x}_1 - \bar{x}_2$	$\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ **or pooled**
12	8.1	Pop proportion	p	Sample prop	$\hat{p}$	???
12	8.2	Diff in pop proportions	$p_1 - p_2$	Diff in sample proportions	$\hat{p}_1 - \hat{p}_2$	???

Goals for today (Sections 8.1-8.2)

- Statistical inference for a single proportion or the difference of two (independent) proportions
  1. Sampling distribution for a proportion or difference in proportions
  2. What are H_0 and H_a?
  3. What are the SE's for $\hat{p}$ and $\hat{p}_1 - \hat{p}_2$?
  4. Hypothesis test
  5. Confidence Interval
  6. How are the SE's different for a hypothesis test & CI?
  7. How to run proportions tests in R
  8. Power & sample size for proportions tests (extra material)

Motivating example

One proportion

- A 2010 study found that out of 269 male college students, 35% had participated in sports betting in the previous year.
  - What is the CI for the proportion?
  - The study also reported that 36% of noncollege young males had participated in sports betting. Is the proportion for male college students different from 0.36?

Two proportions

- There were 214 men in the sample of noncollege young males (36% participated in sports betting in the previous year).
- Compare the difference in proportions between the college and noncollege young males.
  - CI & Hypothesis test

Barnes GM, Welte JW, Hoffman JH, Tidwell MC. Comparisons of gambling and alcohol use among college students and noncollege young people in the United States. J Am Coll Health. 2010 Mar-Apr;58(5):443-52. doi: 10.1080/07448480903540499. PMID: 20304756; PMCID: PMC4104810.

Steps in a Hypothesis Test

1. Set the **level of significance** α
2. Specify the **null** (H_0) and **alternative** (H_A) **hypotheses**
    1. In symbols
    2. In words
    3. Alternative: one- or two-sided?
3. Calculate the **test statistic**.
4. Calculate the **p-value** based on the observed test statistic and its sampling distribution
5. Write a **conclusion** to the hypothesis test
    1. Do we reject or fail to reject H_0?
    2. Write a conclusion in the context of the problem

Step 2: Null & Alternative Hypotheses

Null and alternative hypotheses in **words** and in **symbols**.

One sample test

- H_0: The population proportion of young male college students that participated in sports betting in the previous year is 0.36.
- H_A: The population proportion of young male college students that participated in sports betting in the previous year is not 0.36.

$$H_0 : p = 0.36$$
$$H_A : p \neq 0.36$$

p = population proportion

Two samples test

- H_0: The difference in population proportions of young male college and noncollege students that participated in sports betting in the previous year is 0.
- H_A: The difference in population proportions of young male college and noncollege students that participated in sports betting in the previous year is not 0.

$$H_0 : p_{coll} - p_{noncoll} = 0$$
$$H_A : p_{coll} - p_{noncoll} \neq 0$$

Sampling distribution of $\hat{p}$

- $\hat{p} = \frac{X}{n}$ where X is the number of "successes" and n is the sample size.
- $X \sim Bin(n, p)$, where p is the population proportion.
- For n "big enough", the normal distribution can be used to approximate a binomial distribution:

$$X \sim Bin(n,p) \rightarrow N\left(\mu = np, \sigma = \sqrt{np(1-p)}\right)$$

$$Var\left(\frac{X}{n}\right) = \frac{Var(X)}{n^2}$$

- Since $\hat{p} = \frac{X}{n}$ is a linear transformation of X, we have for large n:

$$\hat{p} \sim N\left(\mu_{\hat{p}} = p, \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}\right)$$

- *How we apply this result to CI's and test statistics is different!!!*

Step 3: Test statistic

Sampling distribution of $\hat{p}$ if we assume $H_0 : p = p_0$ is true:

$$\hat{p} \sim N\left(\mu_{\hat{p}} = p, \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}\right) \sim N\left(\mu_{\hat{p}} = p_0, \sigma_{\hat{p}} = \sqrt{\frac{p_0 \cdot (1-p_0)}{n}}\right)$$

Test statistic for a one sample proportion test:

$$\text{test stat} = \frac{\text{point estimate} - \text{null value}}{SE} = z_{\hat{p}} = \frac{\hat{p} - p_0}{\sqrt{\frac{p_0 \cdot (1-p_0)}{n}}}$$

Example: A 2010 study found that out of 269 male college students, 35% had participated in sports betting in the previous year.

What is the test statistic when testing $H_0 : p = 0.36$ vs. $H_A : p \neq 0.36$?

$\hat{p} = .35 = \frac{x}{269}$

$.35(269) = 94.15$

$$z_{\hat{p}} = \frac{94/269 - 0.36}{\sqrt{\frac{0.36 \cdot (1-0.36)}{269}}} = -0.3607455$$

(.35)

Step "3b": Conditions satisfied?

Conditions:

1. Independent observations?
   - The observations were collected independently.
2. The number of **expected successes and expected failures is at least 10**.
   - $n_1 p_0 \geq 10, \;\; n_1(1-p_0) \geq 10$

Example: A 2010 study found that out of 269 male college students, 35% had participated in sports betting in the previous year.

Testing $H_0: p = 0.36$ vs. $H_A: p \neq 0.36$.

Are the conditions satisfied?

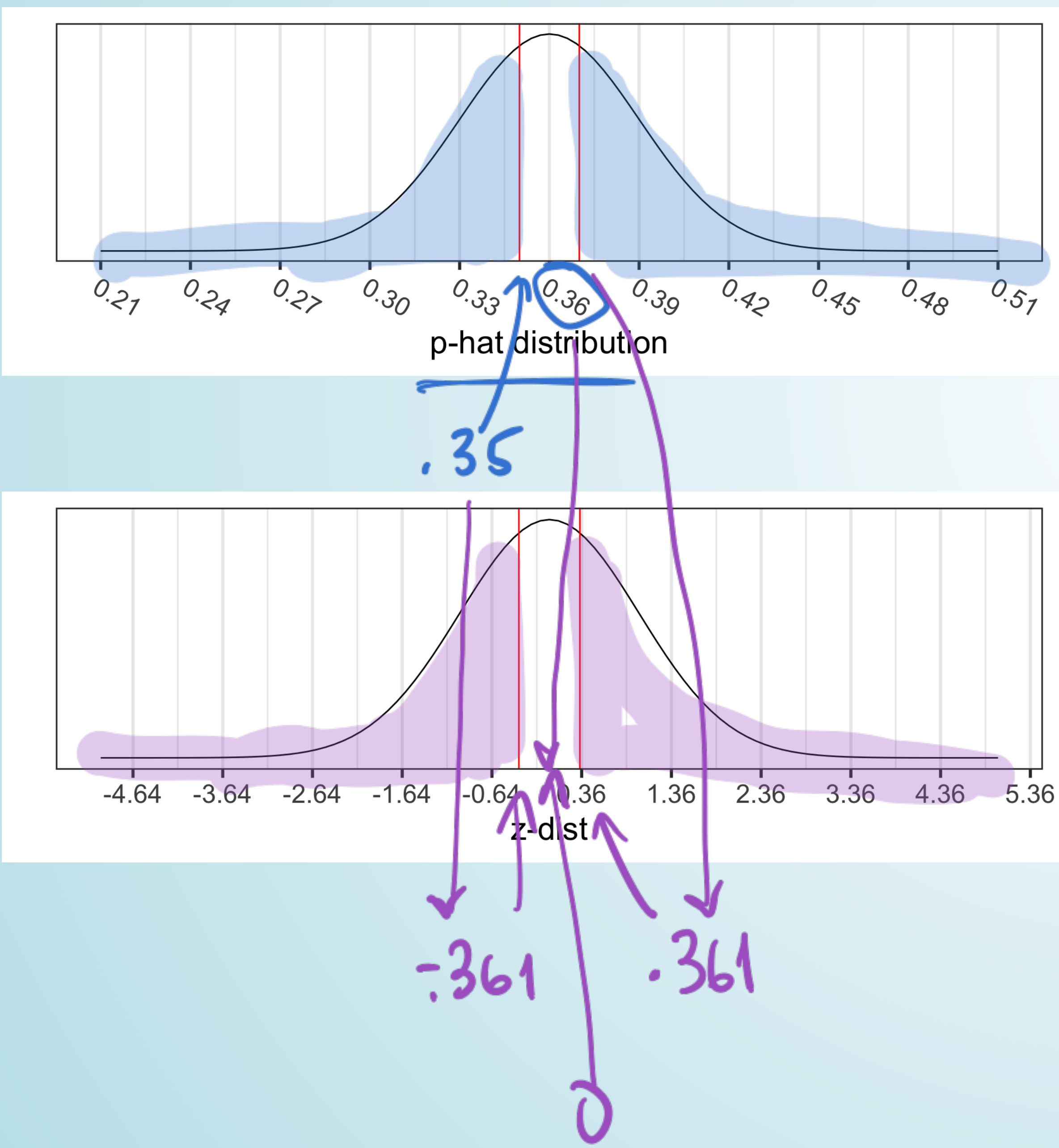
$n_1 p_0 = 269(0.36) = 96.8 \geq 10$ ✓

$n_1(1-p_0) = 269(.64) = 172.2 \geq 10$ ✓

Step 4: p-value

$H_a: p \neq 0.36$

The **p-value** is the **probability** of obtaining a test statistic *just as extreme or more extreme* than the observed test statistic assuming the null hypothesis H_0 is true.



Calculate the *p*-value:

$$2 \cdot P(\hat{p} < 0.35)$$

$$= 2 \cdot P\left(Z_{\hat{p}} < \frac{94/269 - 0.36}{\sqrt{\frac{0.36 \cdot (1-0.36)}{269}}}\right)$$

$$= 2 \cdot P(Z_{\hat{p}} < -0.3607455)$$

$$= 0.7182897$$

```
2*pnorm(-0.3607455)
```

```
[1] 0.7182897
```

Step 5: Conclusion to hypothesis test

$$H_0 : p = 0.36$$
$$H_A : p \neq 0.36$$

- Recall the p-value = 0.7182897 >0.05
- Use α = 0.05.
- Do we reject or fail to reject H_0?

Conclusion statement:

- Stats class conclusion
  - There is insufficient evidence that the (population) proportion of young male college students that participated in sports betting in the previous year is different than 0.36 (p-value = 0.72).
- More realistic manuscript conclusion:
  - In a sample of 269 male college students, 35% had participated in sports betting in the previous year, which is not different from 36% (p-value = 0.72).

95% CI for population proportion

What to use for SE in CI formula?

$$\hat{p} \pm z^* \cdot SE_{\hat{p}}$$

Sampling distribution of $\hat{p}$:

$$\hat{p} \sim N\left(\mu_{\hat{p}} = p, \sigma_{\hat{p}} = \sqrt{\frac{p(1-p)}{n}}\right)$$

Problem: We don't know what p is - it's what we're estimating with the CI.
Solution: approximate p with $\hat{p}$:

$$SE_{\hat{p}} = \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$$

Example: A 2010 study found that out of 269 male college students, 35% had participated in sports betting in the previous year. Find the 95% CI for the population proportion.

qnorm(.975)

$$94/269 \pm 1.96 \cdot SE_{\hat{p}}$$

$$SE_{\hat{p}} = \sqrt{\frac{(94/269)(1-94/269)}{269}}$$

$$(0.293, 0.407)$$

Interpretation:
We are 95% confident that the (population) proportion of young male college students that participated in sports betting in the previous year is in (0.29, 0.41).

Conditions for one proportion: **test vs. CI**

Hypothesis test conditions

1. *Independent observations*
   - The observations were collected independently.

2. The number of **expected** successes and **expected** failures is at least 10.

$$n_1 p_0 \geq 10, \;\; n_1(1 - p_0) \geq 10$$

Confidence interval conditions

1. *Independent observations*
   - The observations were collected independently.

2. The number of successes and failures is at least 10:

$$n_1 \hat{p}_1 \geq 10, \;\; n_1(1 - \hat{p}_1) \geq 10$$

$n_1 \hat{p}_1 = 269(.35) = 94.15 \geq 10$ ✓

$n_1 (1 - \hat{p}_1) = 269(.65) = 174.85 \geq 10$ ✓

Inference for difference of two independent proportions

$\hat{p}_1 - \hat{p}_2$

Sampling distribution of $\hat{p}_1 - \hat{p}_2$

- $\hat{p}_1 = \frac{X_1}{n_1}$ and $\hat{p}_2 = \frac{X_2}{n_2}$,
  - X_1 & X_2 are the number of "successes"
  - n_1 & n_2 are the sample sizes of the 1st & 2nd samples

$\hat{p} \sim Bin(n, p)$

- Each $\hat{p}$ can be approximated by a normal distribution, for "big enough" n
- Since the difference of independent normal random variables is also normal, it follows that for "big enough" n_1 and n_2

$$\hat{p}_1 - \hat{p}_2 \sim N\left(\mu_{\hat{p}_1-\hat{p}_2} = p_1 - p_2,\ \sigma_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{p_1 \cdot (1-p_1)}{n_1} + \frac{p_2 \cdot (1-p_2)}{n_2}}\right)$$

where p_1 & p_2 are the population proportions, respectively.

- *How we apply this result to CI's and test statistics is different!!!*

Step 3: Test statistic (1/2)

Sampling distribution of $\hat{p}_1 - \hat{p}_2$:

$$\hat{p}_1 - \hat{p}_2 \sim N\left(\mu_{\hat{p}_1-\hat{p}_2} = p_1 - p_2,\ \ \sigma_{\hat{p}_1-\hat{p}_2} = \sqrt{\frac{p_1 \cdot (1-p_1)}{n_1} + \frac{p_2 \cdot (1-p_2)}{n_2}}\right)$$

$p_1 = p_2$

Since we assume $H_0 : p_1 - p_2 = 0$ is true, we "pool" the proportions of the two samples to calculate the SE:

$$\text{pooled proportion} = \hat{p}_{pool} = \frac{\text{total number of successes}}{\text{total number of cases}} = \frac{x_1 + x_2}{n_1 + n_2}$$

Test statistic:

$$\text{test statistic} = z_{\hat{p}_1-\hat{p}_2} = \frac{\hat{p}_1 - \hat{p}_2 - 0}{\sqrt{\frac{\hat{p}_{pool}\cdot(1-\hat{p}_{pool})}{n_1} + \frac{\hat{p}_{pool}\cdot(1-\hat{p}_{pool})}{n_2}}}$$

Step 3: Test statistic (2/2)

$$\text{test statistic} = z_{\hat{p}_1-\hat{p}_2} = \frac{\hat{p}_1 - \hat{p}_2 - 0}{\sqrt{\frac{\hat{p}_{pool}\cdot(1-\hat{p}_{pool})}{n_1} + \frac{\hat{p}_{pool}\cdot(1-\hat{p}_{pool})}{n_2}}}$$

$$\text{pooled proportion} = \hat{p}_{pool} = \frac{\text{total number of successes}}{\text{total number of cases}} = \frac{x_1 + x_2}{n_1 + n_2} = \frac{94+77}{269+214} = 0.354$$

Example: A 2010 study found that out of 269 male college students, 35% had participated in sports betting in the previous year, and out of 214 noncollege young males 36% had.
What is the test statistic when testing $H_0 : p_{coll} - p_{noncoll} = 0$ vs. $H_A : p_{coll} - p_{noncoll} \neq 0$?

$$z_{\hat{p}_1-\hat{p}_2} = \frac{94/269 - 77/214 - 0}{\sqrt{0.354 \cdot (1 - 0.354)(\frac{1}{269} + \frac{1}{214})}}$$
$$= -0.2367497$$

Step "3b": Conditions satisfied?

Conditions:

- *Independent observations & samples*
  - The observations were collected independently.
  - In particular, observations from the two groups weren't paired in any meaningful way.
- The number of expected successes and expected failures is at least 10 *for each group* - using the pooled proportion:
  - $n_1\hat{p}_{pool} \geq 10, \quad n_1(1 - \hat{p}_{pool}) \geq 10$
  - $n_2\hat{p}_{pool} \geq 10, \quad n_2(1 - \hat{p}_{pool}) \geq 10$

$\hat{p}_{pool} = 0.354$

Example: A 2010 study found that out of 269 male college students, 35% had participated in sports betting in the previous year, and out of 214 noncollege young males 36% had.
Testing $H_0 : p_{coll} - p_{noncoll} = 0$ vs. $H_A : p_{coll} - p_{noncoll} \neq 0$? .

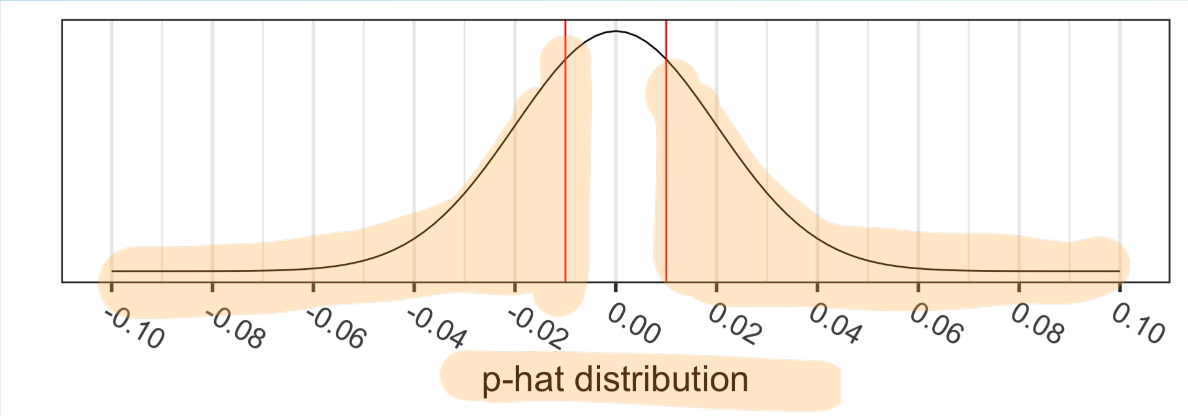
Are the conditions satisfied?

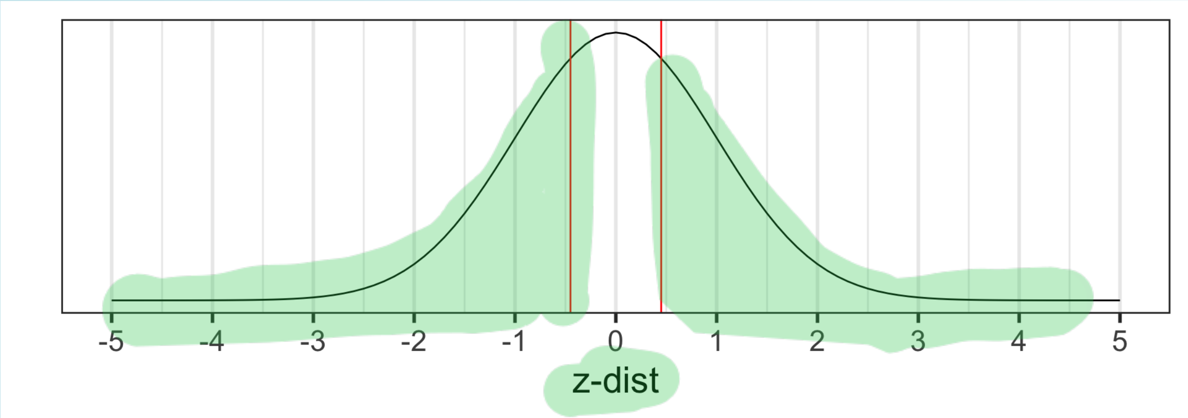
$269(0.354) = 95.226 \geq 10 \quad 269(0.646) = 173.774 \geq 10$

$214(0.354) = 75.756 \geq 10 \quad 214(0.646) = 138.244 \geq 10$

Step 4: p-value

The **p-value** is the **probability** of obtaining a test statistic *just as extreme or more extreme* than the observed test statistic assuming the null hypothesis H_0 is true.





Calculate the *p*-value:

$$2 \cdot P(\hat{p}_1 - \hat{p}_2 < 0.35 - 0.36)$$

$$= 2 \cdot P\left(Z_{\hat{p}_1 - \hat{p}_2} < \frac{94/269 - 77/214 - 0}{\sqrt{0.354 \cdot (1 - 0.354)(\frac{1}{269} + \frac{1}{214})}}\right)$$

$$= 2 \cdot P(Z_{\hat{p}} < -0.2367497)$$

$$= 0.812851$$

```
2*pnorm(-0.2367497)
```

```
[1] 0.812851
```

Step 5: Conclusion to hypothesis test

$$H_0: p_{coll} - p_{noncoll} = 0$$
$$H_A: p_{coll} - p_{noncoll} \neq 0$$

- Recall the p-value = 0.812851
- Use α = 0.05.
- Do we reject or fail to reject H_0?

Conclusion statement:

- Stats class conclusion
  - There is insufficient evidence that the difference in (population) proportions of young male college and noncollege students that participated in sports betting in the previous year are different (p-value = 0.81).
- More realistic manuscript conclusion:
  - 35% of young male college students (n=269) and 36% of noncollege young males (n=214) participated in sports betting in the previous year (p-value = 0.81).

95% CI for population difference in proportions

What to use for SE in CI formula?

$$\hat{p}_1 - \hat{p}_2 \pm z^* \cdot SE_{\hat{p}_1 - \hat{p}_2}$$

SE in sampling distribution of $\hat{p}_1 - \hat{p}_2$

$$\sigma_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{p_1 \cdot (1 - p_1)}{n_1} + \frac{p_2 \cdot (1 - p_2)}{n_2}}$$

Problem: We don't know what p is - it's what we're estimating with the CI.
Solution: approximate p_1, p_2 with $\hat{p}_1$, $\hat{p}_2$:

$$SE_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{\hat{p}_1 \cdot (1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2 \cdot (1 - \hat{p}_2)}{n_2}}$$

Example: A 2010 study found that out of 269 male college students, 35% had participated in sports betting in the previous year, and out of 214 noncollege young males 36% had. Find the 95% CI for the difference in population proportions.

$$\frac{94}{269} - \frac{77}{214} \pm 1.96 \cdot SE_{\hat{p}_1 - \hat{p}_2}$$

$$SE_{\hat{p}_1 - \hat{p}_2} = \sqrt{\frac{94/269 \cdot (1 - 94/269)}{269} + \frac{77/214 \cdot (1 - 77/214)}{214}}$$

Interpretation:
We are 95% confident that the difference in (population) proportions of young male college and noncollege students that participated in sports betting in the previous year is in (-0.127, 0.106).

Conditions for difference in proportions: test vs. CI

Hypothesis test conditions

1. *Independent observations & samples*
   - The observations were collected independently.
   - In particular, observations from the two groups weren't paired in any meaningful way.

2. The number of **expected** successes and **expected** failures is at least 10 *for each group* - using the pooled proportion:
   - $n_1\hat{p}_{pool} \geq 10, \;\; n_1(1-\hat{p}_{pool}) \geq 10$
   - $n_2\hat{p}_{pool} \geq 10, \;\; n_2(1-\hat{p}_{pool}) \geq 10$

Confidence interval conditions

1. *Independent observations & samples*
   - The observations were collected independently.
   - In particular, observations from the two groups weren't paired in any meaningful way.

2. The number of successes and failures is at least 10 *for each group*.
   - $n_1\hat{p}_1 \geq 10, \;\; n_1(1-\hat{p}_1) \geq 10$
   - $n_2\hat{p}_2 \geq 10, \;\; n_2(1-\hat{p}_2) \geq 10$

$269(94/269) = 94 \geq 10 \qquad 269(175/269) = 175 \geq 10$

$214(77/214) = 77 \geq 10 \qquad 214(137/214) = 137 \geq 10$

R: 1- and 2-sample proportions tests

```
prop.test(x, n, p = NULL,
   alternative = c("two.sided", "less", "greater"),
   conf.level = 0.95,
   correct = TRUE)
```

- 2 options for data input
  1. Summary counts
     - x = vector with counts of “successes”
     - n = vector with sample size in each group
  2. Dataset
     - `x = table()` of dataset
     - Need to create a dataset based on the summary stats if do not already have one
- Continuity correction

R: 1-sample proportion test

"1-prop z-test"

Summary stats input for 1-sample proportion test

Example: A 2010 study found that out of 269 male college students, 35% had participated in sports betting in the previous year.
Test $H_0: p = 0.36$ vs. $H_A: p \neq 0.36$?

```
.35*269 # number of "successes"; round this value
```

```
[1] 94.15
```

```
prop.test(x = 94, n = 269,             # x = # successes & n = sample size
          p = 0.36,                    # null value p0
          alternative = "two.sided",   # 2-sided alternative
          correct = FALSE)             # no continuity correction
```

```

	1-sample proportions test without continuity correction

data:  94 out of 269, null probability 0.36
X-squared = 0.13014, df = 1, p-value = 0.7183
alternative hypothesis: true p is not equal to 0.36
95 percent confidence interval:
 0.2949476 0.4081767
sample estimates:
        p 
0.3494424 
```

$\chi^2_{df=1} \sim Z^2$ $Z \sim N(0,1)$

$\chi^2 = 0.13014 = z^2$

$z = \sqrt{z^2} = \sqrt{0.13014} = 0.3607$

Can `tidy()` test output:

```
prop.test(x = 94, n = 269, p = 0.36, alternative = "two.sided", correct = FALSE) %>%
  tidy() %>% gt()
```

estimate	statistic	p.value	parameter	conf.low	conf.high	method	alternative
0.3494424	0.1301373	0.7182897	1	0.2949476	0.4081767	1-sample proportions test without continuity correction	two.sided

Dataset input for 1-sample proportion test (1/2)

Since we don't have a dataset, we first need to create a dataset based on the results:

> "out of 269 male college students, 35% had participated in sports betting in the previous year"

```
.35*269 # number of "successes"; roun
```

```
[1] 94.15
```

```
SportsBet1 <- tibble(
  Coll = c(rep("Bet", 94),
           rep("NotBet",269-94))
  )
```

```
glimpse(SportsBet1)
```

```
Rows: 269
Columns: 1
$ Coll <chr> "Bet", "Bet", "Bet", "Bet", "Bet",
"Bet", "Bet", "Bet", "Bet", "B…
```

```
SportsBet1 %>% tabyl(Coll)
```

```
   Coll   n   percent
    Bet  94 0.3494424
 NotBet 175 0.6505576
```

R code for proportions test requires input as a base R `table`:

```
table(SportsBet1$Coll)
```

```

   Bet NotBet 
    94    175 
```

Dataset input for 1-sample proportion test (2/2)

- When using a dataset, `prop.test` requires the input x to be a table
- Note that we do not also specify n since the table already includes all needed information.

```
prop.test(x = table(SportsBet1$Coll),    # table() of data
          p = 0.36,                       # null value p0
          alternative = "two.sided",      # 2-sided alternative
          correct = FALSE)                # no continuity correction
```

```

	1-sample proportions test without continuity correction

data:  table(SportsBet1$Coll), null probability 0.36
X-squared = 0.13014, df = 1, p-value = 0.7183
alternative hypothesis: true p is not equal to 0.36
95 percent confidence interval:
 0.2949476 0.4081767
sample estimates:
        p
0.3494424
```

Compare output with summary stats method:

```
prop.test(x = 94, n = 269,               # x = # successes & n = sample size
          p = 0.36,                       # null value p0
          alternative = "two.sided",      # 2-sided alternative
          correct = FALSE)  %>%           # no continuity correction
  tidy() %>% gt()
```

estimate	statistic	p.value	parameter	conf.low	conf.high	method	alternative
0.3494424	0.1301373	0.7182897	1	0.2949476	0.4081767	1-sample proportions test without continuity correction	two.sided

Continuity correction: 1-prop z-test with vs. without CC

- Recall that when we approximated the
- binomial distribution with a normal distribution to calculate a probability,
- that we included a continuity correction (CC)
- to account for approximating a discrete distribution with a continuous distribution.

```
prop.test(x = 94, n = 269, p = 0.36, alternative = "two.sided",
          correct = FALSE) %>%
  tidy() %>% gt()
```

estimate	statistic	p.value	parameter	conf.low	conf.high	method	alternative
0.3494424	0.1301373	0.7182897	1	0.2949476	0.4081767	1-sample proportions test without continuity correction	two.sided

```
prop.test(x = 94, n = 269, p = 0.36, alternative = "two.sided",
          correct = TRUE) %>%
  tidy() %>% gt()
```

estimate	statistic	p.value	parameter	conf.low	conf.high	method	alternative
0.3494424	0.08834805	0.7662879	1	0.2931841	0.4100774	1-sample proportions test with continuity correction	two.sided

Differences are small when sample sizes are large.

R: 2-samples proportion test

"2-prop z-test"

Summary stats input for 2-samples proportion test

Example: A 2010 study found that out of 269 male college students, 35% had participated in sports betting in the previous year, and out of 214 noncollege young males 36% had. Test $H_0 : p_{coll} - p_{noncoll} = 0$ vs. $H_A : p_{coll} - p_{noncoll} \neq 0$.

```
# round the number of successes:
.35*269 # number of "successes" in college students
```

```
[1] 94.15
```

```
.36*214 # number of "successes" in noncollege students
```

```
[1] 77.04
```

```
NmbrBet <- c(94, 77)                   # vector for # of successes in each group
TotalNmbr <- c(269, 214)               # vector for sample size in each group

prop.test(x = NmbrBet,                 # x is # of successes in each group
          n = TotalNmbr,               # n is sample size in each group
          alternative = "two.sided",   # 2-sided alternative
          correct = FALSE)             # no continuity correction
```

```
	2-sample test for equality of proportions without continuity correction

data:  NmbrBet out of TotalNmbr
X-squared = 0.05605, df = 1, p-value = 0.8129
alternative hypothesis: two.sided
95 percent confidence interval:
 -0.09628540  0.07554399
sample estimates:
   prop 1    prop 2
0.3494424 0.3598131
```

$z = \sqrt{0.05605} = 0.2367$

Dataset input for 2-samples proportion test (1/2)

Since we don't have a dataset, we first need to create a dataset based on the results:

> "out of 269 male college students, 35% had participated in sports betting in the previous year, and out of 214 noncollege young males 36% had"

```
# round the number of successes:
.35*269 # college students
```

```
[1] 94.15
```

```
.36*214 # noncollege students
```

```
[1] 77.04
```

```
SportsBet2 <- tibble(
  Group = c(rep("College", 269),
            rep("NonCollege", 214)),
  Bet = c(rep("yes", 94),
          rep("no", 269-94),
          rep("yes", 77),
          rep("no", 214-77))
)
```

```
glimpse(SportsBet2)
```

```
Rows: 483
Columns: 2
$ Group <chr> "College", "College", "College",
"College", "College", "College"…
$ Bet   <chr> "yes", "yes", "yes", "yes", "yes",
"yes", "yes", "yes", "yes", "…
```

```
SportsBet2 %>% tabyl(Group, Bet)
```

```
      Group  no yes
    College 175  94
 NonCollege 137  77
```

R code for proportions test requires input as a base R `table`:

```
table(SportsBet2$Group,
      SportsBet2$Bet)
```

```
              no yes
  College    175  94
  NonCollege 137  77
```

Dataset input for 2-samples proportion test (2/2)

- When using a dataset, `prop.test` requires the input `x` to be a table
- Note that we do not also specify `n` since the table already includes all needed information.

```
prop.test(x = table(SportsBet2$Group, SportsBet2$Bet),
        alternative = "two.sided",
        correct = FALSE)
```

```

	2-sample test for equality of proportions without continuity correction

data:  table(SportsBet2$Group, SportsBet2$Bet)
X-squared = 0.05605, df = 1, p-value = 0.8129
alternative hypothesis: two.sided
95 percent confidence interval:
 -0.07554399  0.09628540
sample estimates:
   prop 1    prop 2
0.6505576 0.6401869
```

} proportion "no"

"no" is before "yes" in alphanumeric order

Compare output with summary stats method:

```
prop.test(x = NmbrBet,                    # x is # of successes in each group
          n = TotalNmbr,                  # n is sample size in each group
          alternative = "two.sided",      # 2-sided alternative
          correct = FALSE) %>%            # no continuity correction
  tidy() %>% gt()
```

estimate1	estimate2	statistic	p.value	parameter	conf.low	conf.high	method	alternative
0.3494424	0.3598131	0.05605044	0.8128509	1	-0.0962854	0.07554399	2-sample test for equality of proportions without continuity correction	two.sided

proportion "yes"

Continuity correction: 2-prop z-test with vs. without CC

- Recall that when we approximated the
- binomial distribution with a normal distribution to calculate a probability,
- that we included a continuity correction (CC)
- to account for approximating a discrete distribution with a continuous distribution.

```
prop.test(x = NmbrBet, n = TotalNmbr, alternative = "two.sided",
          correct = FALSE) %>% tidy() %>% gt()
```

estimate1	estimate2	statistic	p.value	parameter	conf.low	conf.high	method	alternative
0.3494424	0.3598131	0.05605044	0.8128509	1	-0.0962854	0.07554399	2-sample test for equality of proportions without continuity correction	two.sided

```
prop.test(x = NmbrBet, n = TotalNmbr, alternative = "two.sided",
          correct = TRUE) %>% tidy() %>% gt()
```

estimate1	estimate2	statistic	p.value	parameter	conf.low	conf.high	method	alternative
0.3494424	0.3598131	0.01987511	0.8878864	1	-0.1004806	0.07973918	2-sample test for equality of proportions with continuity correction	two.sided

Differences are small when sample sizes are large.

Power & sample size for testing proportions

Sample size calculation for testing one proportion

- Recall in our sports betting example that the null $p_0 = 0.36$ and the observed proportion was $\hat{p} = 0.35$.
  - The *p*-value from the hypothesis test was not significant.
  - How big would the sample size n need to be in order for the *p*-value to be significant?
- **Calculate n**
  - given α, power ($1 - \beta$), "true" alternative proportion p, and null p_0:

$$n = p(1-p)\left(\frac{z_{1-\alpha/2} + z_{1-\beta}}{p - p_0}\right)^2$$

```
p <- 0.35
p0 <- 0.36
alpha <- 0.05
beta <- 0.20  #power=1-beta; want >=80% power
n <- p*(1-p)*((qnorm(1-alpha/2) + qnorm(1-beta)) /
                    (p-p0))^2
n
```

```
[1] 17856.2
```

```
ceiling(n)
```

```
[1] 17857
```

We would need a sample size of at least 17,857!

Power calculation for testing one proportion

Conversely, we can calculate how much power we had in our example given the sample size of 269.

- **Calculate power**,
  - given α, n, "true" alternative proportion p, and null p_0

$$1-\beta = \Phi\left(z - z_{1-\alpha/2}\right) + \Phi\left(-z - z_{1-\alpha/2}\right) \quad , \quad \text{where } z = \frac{p-p_0}{\sqrt{\frac{p(1-p)}{n}}}$$

Φ is the probability for a standard normal distribution

```r
p <- 0.35; p0 <- 0.36; alpha <- 0.05; n <- 269
(z <- (p-p0)/sqrt(p*(1-p)/n))
```

```
[1] -0.343863
```

```r
(Power <- pnorm(z - qnorm(1-alpha/2)) +  pnorm(-z - qnorm(1-alpha/2)))
```

```
[1] 0.06365242
```

If the population proportion is 0.35 instead of 0.36, we only have a 6.4% chance of correctly rejecting H_0 when the sample size is 269.

R package pwr for power analyses

- Specify all parameters *except for* the one being solved for.
- One proportion

```
pwr.p.test(h = NULL, n = NULL, sig.level = 0.05, power = NULL,   alternative
= c("two.sided","less","greater"))
```

- Two proportions (same sample sizes)

```
pwr.2p.test(h = NULL, n = NULL, sig.level = 0.05, power = NULL,
alternative = c("two.sided","less","greater"))
```

- Two proportions (different sample sizes)

```
pwr.2p2n.test(h = NULL, n1 = NULL, n2 = NULL, sig.level = 0.05, power =
NULL,   alternative = c("two.sided", "less","greater"))
```

---

h is the effect size, and calculated using an arcsine transformation:

$$h = \text{ES.h(p1, p2)} = 2\arcsin(\sqrt{p_1}) - 2\arcsin(\sqrt{p_2})$$

See PASS documentation for

- testing 1 proportion using effect size vs. other ways of powering a test of 1 proportion
- testing 2 proportions using effect size vs. other ways of powering a test of 2 proportions.

pwr: sample size for one proportion test

`pwr.p.test(h = NULL, n = NULL, sig.level = 0.05, power = NULL, alternative = c("two.sided","less","greater"))`

- h is the effect size: `h = ES.h(p1, p2)`
  - `p1` and `p2` are the two proportions being tested
  - one of them is the null proportion p_0, and the other is the alternative proportion

Specify all parameters *except for* the sample size:

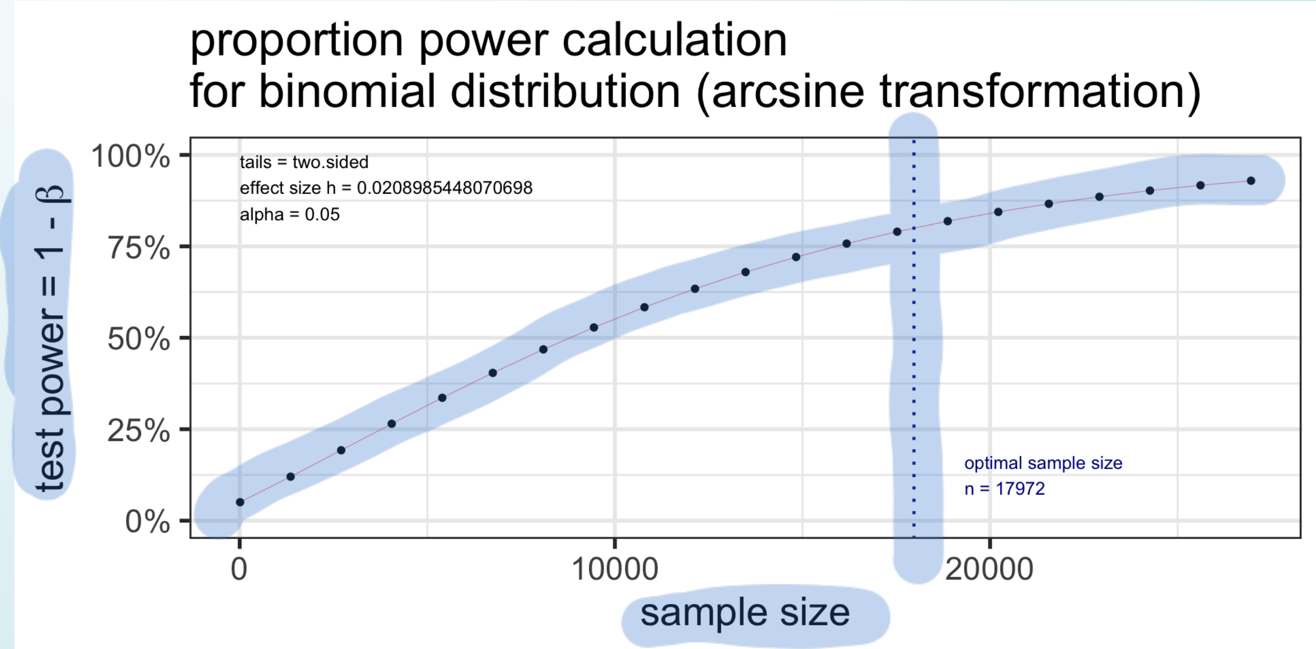
```r
library(pwr)

p.n <- pwr.p.test(
  h = ES.h(p1 = 0.36, p2 = 0.35),
  sig.level = 0.05,
  power = 0.80,
  alternative = "two.sided")
p.n
```

```
     proportion power calculation for binomial
distribution (arcsine transformation)

              h = 0.02089854
              n = 17971.09
      sig.level = 0.05
          power = 0.8
    alternative = two.sided
```

```r
plot(p.n)
```



pwr: power for one proportion test

`pwr.p.test(h = NULL, n = NULL, sig.level = 0.05, power = NULL,   alternative = c("two.sided","less","greater"))`

- h is the effect size: `h = ES.h(p1, p2)`
  - `p1` and `p2` are the two proportions being tested
  - one of them is the null proportion p_0, and the other is the alternative proportion

Specify all parameters *except for* the power:

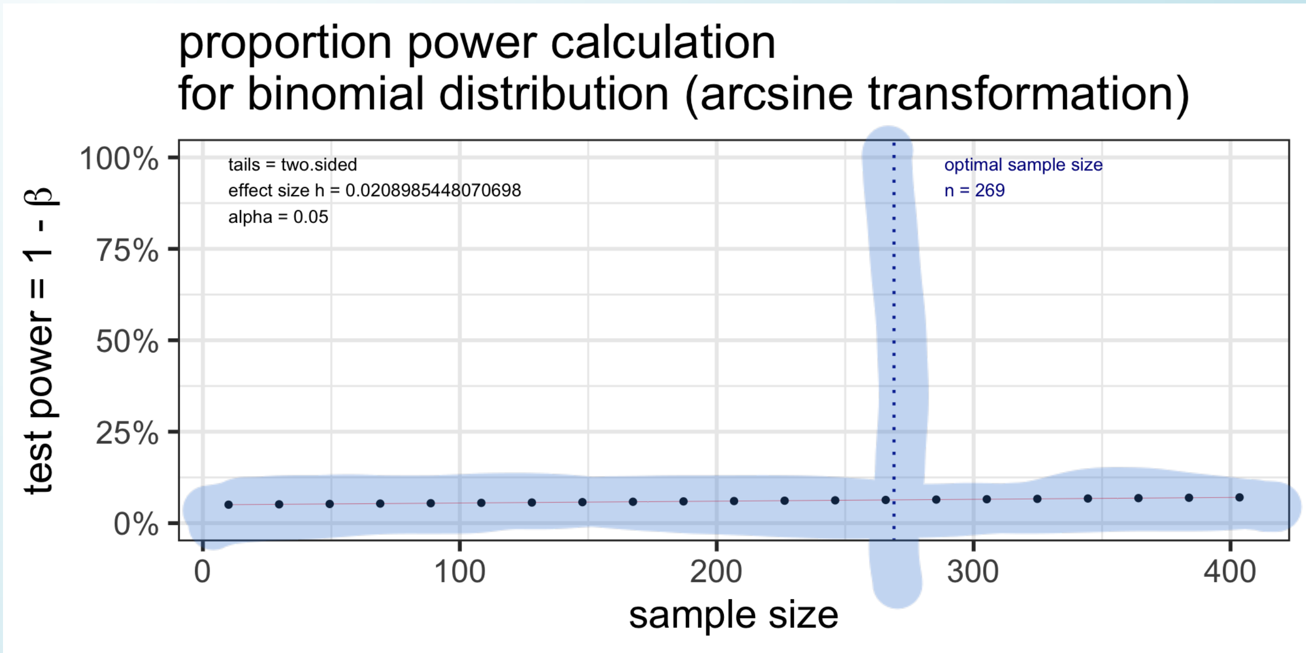
```r
library(pwr)

p.power <- pwr.p.test(
  h = ES.h(p1 = 0.36, p2 = 0.35),
  sig.level = 0.05,
  # power = 0.80,
  n = 269,
  alternative = "two.sided")
p.power
```

```
     proportion power calculation for binomial
distribution (arcsine transformation)

              h = 0.02089854
              n = 269
      sig.level = 0.05
          power = 0.06356445
    alternative = two.sided
```

```r
plot(p.power)
```



pwr: sample size for two proportions test

- Two proportions (same sample sizes)

```
pwr.2p.test(h = NULL, n = NULL, sig.level = 0.05, power = NULL,
alternative = c("two.sided","less","greater"))
```

- h is the effect size: `h = ES.h(p1, p2)`; `p1` and `p2` are the two proportions being tested

Specify all parameters *except for* the sample size:

```
p2.n <- pwr.2p.test(
  h = ES.h(p1 = 0.36, p2 = 0.35),
  sig.level = 0.05,
  power = 0.80,
  alternative = "two.sided")
p2.n
```

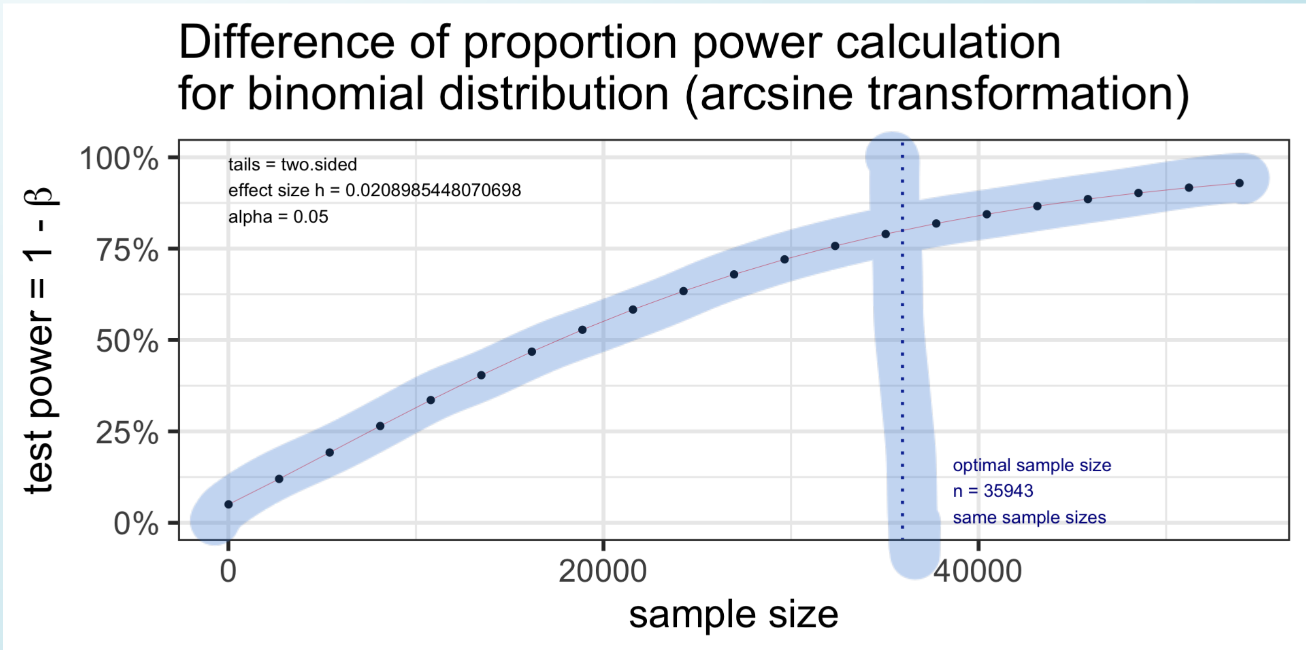
```
     Difference of proportion power calculation
for binomial distribution (arcsine transformation)

              h = 0.02089854
              n = 35942.19
      sig.level = 0.05
          power = 0.8
    alternative = two.sided

NOTE: same sample sizes
```

Note: n in output is the **number per sample**!

```
plot(p2.n)
```



pwr: power for two proportions test

- Two proportions (different sample sizes)

```
pwr.2p2n.test(h = NULL, n1 = NULL, n2 = NULL, sig.level = 0.05, power =
NULL,   alternative = c("two.sided", "less","greater"))
```

- h is the effect size: `h = ES.h(p1, p2)`; `p1` and `p2` are the two proportions being tested

Specify all parameters *except for* the power:

```r
p2.n2 <- pwr.2p2n.test(
  h = ES.h(p1 = 0.36, p2 = 0.35),
  n1 = 214,
  n2 = 269,
  sig.level = 0.05,
  # power = 0.80,
  alternative = "two.sided")
p2.n2
```

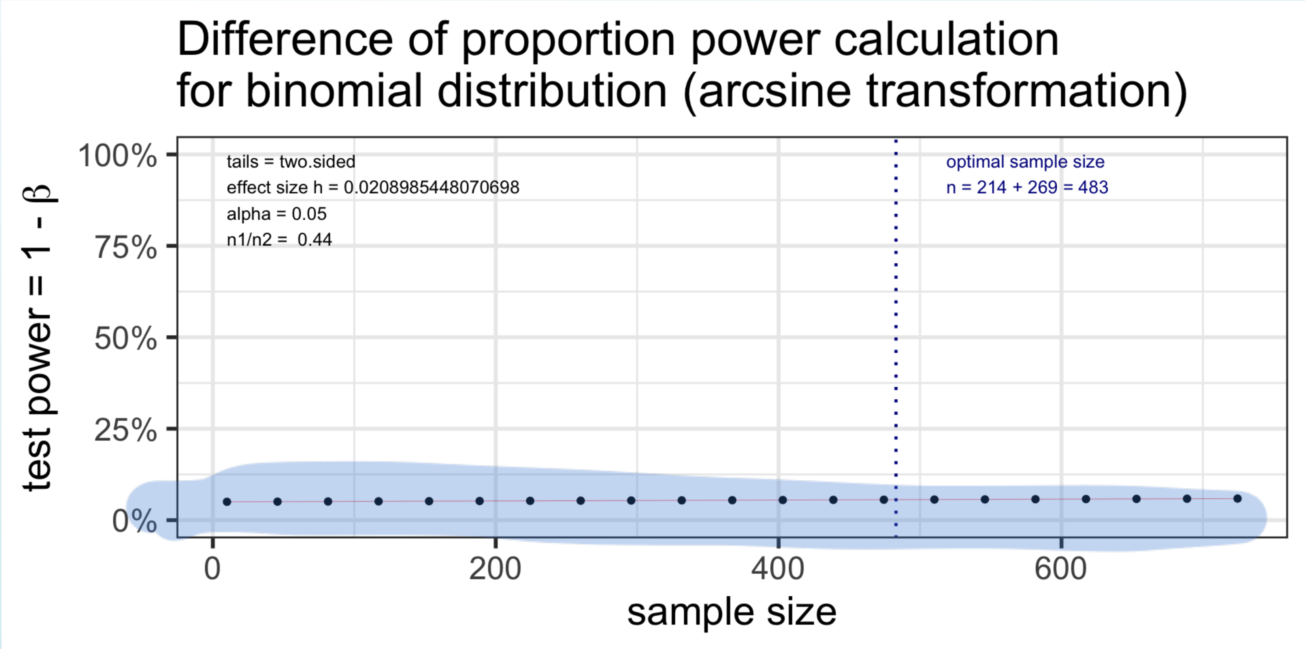
```
     difference of proportion power calculation
 for binomial distribution (arcsine transformation)

              h = 0.02089854
             n1 = 214
             n2 = 269
      sig.level = 0.05
          power = 0.05598413
    alternative = two.sided

NOTE: different sample sizes
```

Note: n in output is the **number per sample**!

```r
plot(p2.n2)
```



Where are we?

* See notes for what to plugin for p, p1, and p2.

CI's and hypothesis tests for different scenarios: Different for CI & hypothesis test!

$$\text{point estimate} \pm z^*(or\ t^*) \cdot SE, \quad \text{test stat} = \frac{\text{point estimate} - \text{null value}}{SE}$$

Day	Book	Population parameter	Symbol	Point estimate	Symbol	SE
10	5.1	Pop mean	μ	Sample mean	$\bar{x}$	$\frac{s}{\sqrt{n}}$
10	5.2	Pop mean of paired diff	μ_d or δ	Sample mean of paired diff	$\bar{x}_d$	$\frac{s_d}{\sqrt{n}}$
11	5.3	Diff in pop means	$\mu_1 - \mu_2$	Diff in sample means	$\bar{x}_1 - \bar{x}_2$	$\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ **or pooled**
12	8.1	Pop proportion	p	Sample prop	$\widehat{p}$	$\sqrt{\frac{p(1-p)}{n}}$ *
12	8.2	Diff in pop proportions	$p_1 - p_2$	Diff in sample proportions	$\widehat{p}_1 - \widehat{p}_2$	$\sqrt{\frac{p_1 \cdot (1-p_1)}{n_1} + \frac{p_2 \cdot (1-p_2)}{n_2}}$ *