Chapter 2: Probability (Part 1)

Example 2.1. ***Rolling fair 6-sided dice***

(1) *Suppose you roll a fair 6-sided die.*
   (a) *What are all the possible outcomes?*

   (b) *What is the probability that you roll a 4?*

   (c) *What is the probability that you roll an even number?*

   (d) *What is the probability that you roll an even number and a 2?*

   (e) *What is the probability that you roll a 2 and a 5?*

   (f) *What is the probability that you did not roll a 3?*

(2) *Suppose you roll* ***two*** *fair 6-sided dice.*
   (a) *What is the probability that you roll a 2 with the first die and a 5 with the second die?*

   (b) *What is the probability that the sum of the two dice is 10?*

2.1. What is a probability?

Definition 2.2. *The **probability** of an outcome is the proportion of times the outcome would occur if the random phenomenon could be observed an infinite number of times.*

Law of Large Numbers
As more observations are collected, the proportion of occurrences with a particular outcome converges to the probability p of that outcome.

2.2. Probabilities of Equally Likely Outcomes.

2.3. **Probability Definitions and Rules.**

1. The **sample space** S is the set of all possible outcomes.

2. **Events** are sets or collections of outcomes.

3. **Disjoint** or **mutually exclusive** events
   Events A and B are disjoint (or, mutually exclusive) if

4. **Addition Rule for Disjoint Events**
   If A and B are disjoint events, then the probability that at least one of them will occur is

5. **General Addition Rule**
   If A and B are any two events, disjoint or not, then the probability that at least one of them will occur is

6. **Complement**
   The complement of event A is denoted A^C, and A^C represents all outcomes not in A.

Example 2.3. ***Diabetes and hypertension.***

Diabetes and hypertension are two of the most common diseases in Western, industrialized nations. In the United States, approximately 9% of the population have diabetes, while about 30% of adults have high blood pressure. The two diseases frequently occur together: an estimated 6% of the population have both diabetes and hypertension.

Let D represent the event of having diabetes, and H the event of having hypertension.

(1) *Are the events having hypertension and having diabetes mutually exclusive?*

(2) *Draw a Venn diagram summarizing the variables and their associated probabilities.*

(3) *Calculate the probability of having diabetes or hypertension.*

(4) *What percent of Americans have neither hypertension nor diabetes?*

Example 2.4. *Pulling cards from a standard deck of cards*

(1) *Supposed you randomly pull 5 cards one after another from a standard deck of cards,*

(a) ***replacing*** *a card before pulling another one. What is the probability of pulling the sequence of* ♢, ♢, ♠, ♡, ♠*?*

(b) ***without replacing*** *any of the cards. What is the probability of pulling the sequence of* ♢, ♢, ♠, ♡, ♠*?*

(2) *Supposed you pull a red card from the deck. What is the probability that it is a heart?*

2.4. More Probability Rules.

1. **General Multiplication Rule**
   For events A and B,
   $$\mathbb{P}(A \text{ and } B) =$$

2. **Conditional Probability Definition**
   The conditional probability of an event A given an event or condition B is
   $$\mathbb{P}(A|B) =$$

3. If A and B are **independent** events, then
   (a)
   $$\mathbb{P}(A|B) =$$
   (b)
   $$\mathbb{P}(A \text{ and } B) =$$

4. **Sum of Conditional Probabilities**
   $\mathbb{P}(A|B)$ is a probability, meaning that it satisfies the usual probability rules. In particular,
   $$\mathbb{P}(A|B) + \mathbb{P}(A^C|B) =$$

Example 2.5. ***Diabetes and hypertension revisited.***
Diabetes and hypertension are two of the most common diseases in Western, industrialized nations. In the United States, approximately 9% of the population have diabetes, while about 30% of adults have high blood pressure. The two diseases frequently occur together: an estimated 6% of the population have both diabetes and hypertension.
Let D represent the event of having diabetes, and H the event of having hypertension. Is the event of someone being hypertensive independent of the event that someone has diabetes?

Example 2.6. ***Seat belts***

Seat belt use is the most effective way to save lives and reduce injuries in motor vehicle crashes. In a 2014 survey, respondents were asked, "How often do you use seat belts when you drive or ride in a car?". The following table shows the distribution of seat belt usage by sex.

		Seat Belt Usage					
		Always	*Nearly always*	*Sometimes*	*Seldom*	*Never*	*Total*
Sex	*Male*	*146,018*	*19,492*	*7,614*	*3,145*	*4,719*	*180,988*
	Female	*229,246*	*16,695*	*5,549*	*1,815*	*2,675*	*255,980*
	Total	*375,264*	*36,187*	*13,163*	*4,960*	*7,394*	*436,968*

(1) *Calculate the marginal probability that a randomly chosen individual always wears seatbelts.*

(2) *What is the probability that a randomly chosen female always wears seatbelts?*

(3) *What is the probability of a randomly chosen individual always wearing seatbelts, given that they are female?*
same as previous

(4) *What is the probability of a randomly chosen individual being female and always wearing seatbelts?*

Seat belts cont'd

		Seat Belt Usage					
		Always	*Nearly always*	*Sometimes*	*Seldom*	*Never*	*Total*
Sex	*Male*	*146,018*	*19,492*	*7,614*	*3,145*	*4,719*	*180,988*
	Female	*229,246*	*16,695*	*5,549*	*1,815*	*2,675*	*255,980*
	Total	*375,264*	*36,187*	*13,163*	*4,960*	*7,394*	*436,968*

(5) *What is the probability of a randomly chosen individual always wearing seatbelts, given that they are male?*

(6) *Calculate the probability that an individual who never wears seatbelts is male.*

(7) *Does gender seem independent of seat belt usage?*