Chapter 2: Probability (Part 2)

Example 2.7. ***How accurate is rapid testing for COVID-19?***
From the iHealth® website
`https://ihealthlabs.com/pages/ihealth-covid-19-antigen-rapid-test-details:`
"Based on the results of a clinical study where the iHealth® COVID-19 Antigen Rapid Test was compared to an FDA authorized molecular SARS-CoV-2 test, iHealth® COVID-19 Antigen Rapid Test correctly identified 94.3% of positive specimens and 98.1% of negative specimens."

Suppose you take the iHealth® rapid test.

(1) *What is the probability of a positive test result?*
(2) *What is the probability of having COVID-19 if you get a positive test result?*
(3) *What is the probability of not having COVID-19 if you get a negative test result?*

What information were we given?

First, let's define our events of interest:

- D = *event one has disease (COVID-19)*
- D^c = *event one does not have disease*
- T^+ = *event one tests positive for disease*
- T^- = *event one tests negative for disease*

Translate given information into mathematical notation:

- *Test correctly gives a positive result 94.3% of the time:*

- *Test correctly gives a negative result 98.1% of the time:*

- *What are all the possible scenarios of test results?*

Given: $\mathbb{P}(T^{+}|D) = 0.943, \;\; \mathbb{P}(T^{-}|D^{c}) = 0.981$

Solutions to questions

(1) *What is the probability of a positive test result?*

Given: $\mathbb{P}(T^+|D) = 0.943, \;\; \mathbb{P}(T^-|D^c) = 0.981$

(2) *What is the probability of having COVID-19 if you get a positive test result?*

Given: $\mathbb{P}(T^+|D) = 0.943, \;\; \mathbb{P}(T^-|D^c) = 0.981$

(3) *What is the probability of not having COVID-19 if you get a negative test result?*

Bayes' Theorem (Section 2.2.5)

In the previous examples we derived the formula for Bayes' Theorem.

Theorem 2.8 (Bayes' Theorem). *If the sample space S can be split into disjoint events $A_1, A_2, ..., A_k$ that make up all possible outcomes in S, and if $\mathbb{P}(A_i) > 0$ for $i = 1, \ldots, k$ and $\mathbb{P}(B) > 0$, then*

$$\mathbb{P}(A_1|B) = \frac{\mathbb{P}(B|A_1) \cdot \mathbb{P}(A_1)}{\mathbb{P}(B|A_1) \cdot \mathbb{P}(A_1) + \mathbb{P}(B|A_2) \cdot \mathbb{P}(A_2) + ... + \mathbb{P}(B|A_k) \cdot \mathbb{P}(A_k)}$$

Special case of Bayes' Theorem for sample space being split into A and A^c:

$$\mathbb{P}(A|B) = \frac{\mathbb{P}(B|A) \cdot \mathbb{P}(A)}{\mathbb{P}(B|A) \cdot \mathbb{P}(A) + \mathbb{P}(B|A^c) \cdot \mathbb{P}(A^c)}$$

Theorem 2.9 (Law of Total Probability). *(denominator of Bayes' Theorem)*

If the sample space S can be split into disjoint events $A_1, A_2, ..., A_k$ that make up all possible outcomes in S, and if $\mathbb{P}(A_i) > 0$ for $i = 1, \ldots, k$ and $\mathbb{P}(B) > 0$, then

$$\begin{aligned} \mathbb{P}(B) &= \mathbb{P}(B \textit{ and } A_1) + \mathbb{P}(B \textit{ and } A_2) + \ldots + \mathbb{P}(B \textit{ and } A_k) \\ &= \mathbb{P}(B|A_1) \cdot \mathbb{P}(A_1) + \mathbb{P}(B|A_2) \cdot \mathbb{P}(A_2) + \ldots + \mathbb{P}(B|A_k) \cdot \mathbb{P}(A_k) \end{aligned}$$

Special case of Law of Total Probability for sample space being split into A and A^c:

$$\begin{aligned} \mathbb{P}(B) &= \mathbb{P}(B \text{ and } A) + \mathbb{P}(B \text{ and } A^C) \\ &= \mathbb{P}(B|A) \cdot \mathbb{P}(A) + \mathbb{P}(B|A^C) \cdot \mathbb{P}(A^C) \end{aligned}$$

Example 2.10. ***Antibody test for COVID-19***

According to the FDA's EUA Authorized Serology Test Performance website, the Abbott AdviseDx SARS-CoV-2 IgG II (Alinity) antibody test for COVID-19 has sensitivity 98.1% and PPV 98.4% when the prevalence is 20%.

Question: What is the specificity of the antibody test?

What information were we given?

First, let's define our events of interest:

- A = *event one has antibodies for COVID-19*
- A^c = *event one does not have antibodies*
- T^+ = *event one tests positive for antibodies*
- T^- = *event one tests negative for antibodies*

Translate given information into mathematical notation:

- *Sensitivity is 98.1%:*
- *PPV is 98.4%:*
- *Prevalence is 20%:*

Solution: